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The Resolver as Mechatronical System

The resolver is an electromagnetic angle measuring device. It exists of a stator containing two coils which are offset by 90 °. They are linked with the flux of a rotor coil depending on the angle of rotation. Figure 1 shows the construction. The rotor coil produces the flux ϕ, which decomposes in a part for the perpendicular sinus coil (green) and one for the horizontal cosine coil (cyan).

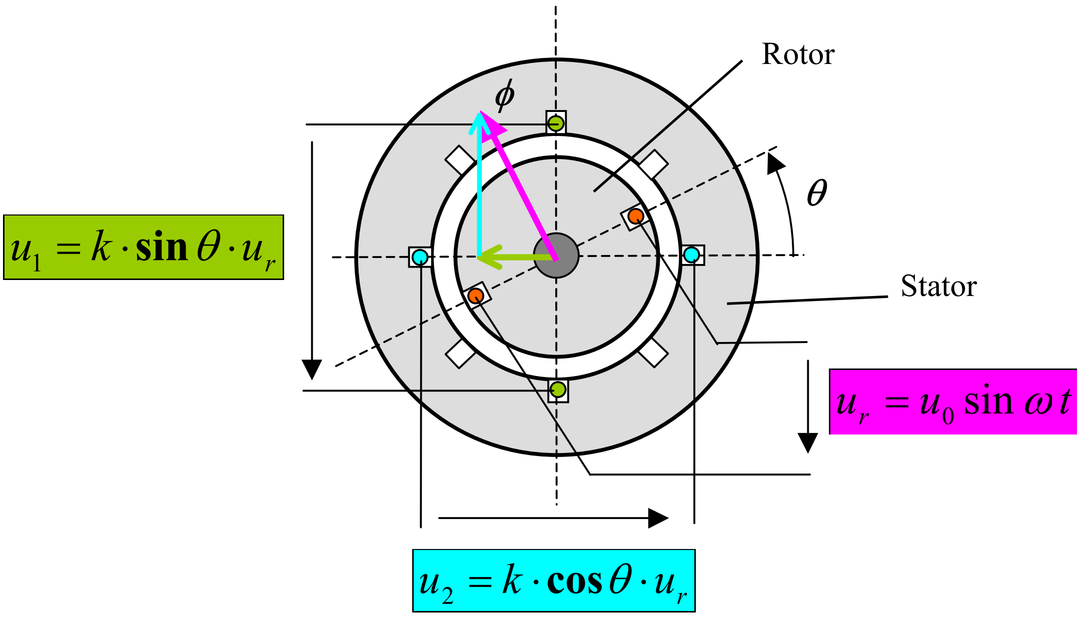

Figure 1: Concept of a resolver

The rotor is fed with the reference voltage $u_r = u_0$ **sin**(ωt). In the perpendicular coil an alternating voltage with the amplitude $\hat{u}_1 = k\, u_0 \,\mathbf{sin}\theta$ is induced and in the horizontal one a voltage with the amplitude $\hat{u}_2 = k\, u_0\, \mathbf{cos}\theta$. k is the coupling faktor between the rotor and the stator coils. From the ratio of the amplitudes $\hat{u}_1$ and $\hat{u}_2$ the angle of rotation can be determined:

$$\frac{\hat{u}_1}{\hat{u}_2} = \frac{k \cdot u_0 \,\mathbf{sin}\,\theta}{k \cdot u_0 \cdot \mathbf{cos}\,\theta} = \mathbf{tan}\,\theta \qquad (1)$$

The immediate result is

$$\theta = \mathbf{arctan}\,\frac{\hat{u}_1}{\hat{u}_2} \qquad (2)$$

Eq. 2 has the drawback though, that the result is ambiguous for $\hat{u}_2 < 0$. This is avoided if the function arctan2 is used instead [1]. Then θ is determined correctly in all four quadrants.

The oparation of the resolver is clear by this. But it also is obvious that the mechanical system from figure 1 has to be complemented by an electronical circuit which delivers the reference voltage u_r and also solves for the angle θ. The functional resolver therefor consits of a mechanical and a special electronical device. The resolvers are supposed to have been develloped at the MIT in 1940 [2].

Many problems can only be solved for application by combining mechanics and electronics. Or by adding electronics the performance of systems can be improved essentially. Therefor the term „mechatronics“ came in use for this discipline of engineering, that combines mechanical engineering and electronis as well as other sciences for the purpose of developing systems with high standards.

Thus the notion „mechatronics“ was build from the two words mechanics und electronics. The notion is believed to be in use since the year 1969. At that time the resolvers were on the market already. Until the year 1982 the Japanise Company Yaskawa Electric Corporation owned the rights on the english term [3, 4]. Later the notion was used all over the world. At the University of Waterloo the following definition is used for the engineering field:

> Mechatronics engineering is the design of computer-controlled electromechanical systems. It can be viewed as 'modern mechanical engineering design' in the sense that the design of the mechanical system must be performed together with that of the electrical/electronic and computer control aspects that will comprise the complete system. [5]

In Germany the first course of studies called „Mechatronik“ was founded in the year 1991, and in 1998 the appranticeship of the „Mechatronic Technician“ was accredited [3].

Evaluation of the Resolver Signals

Though a circuit for the evaluation of equ. 2 for the angle of rotation is possible, it is less useful, however, as the accuracy suffers from the inevitable noise on the signals. In practice a procedure has prevailed that operates as tracking filter. It uses an estimated value of the angle θ^* which is automatically adjusted towards the real angle θ and tracks it if it is changing. Then, of course, a trakking error is possible, which can be kept small however.

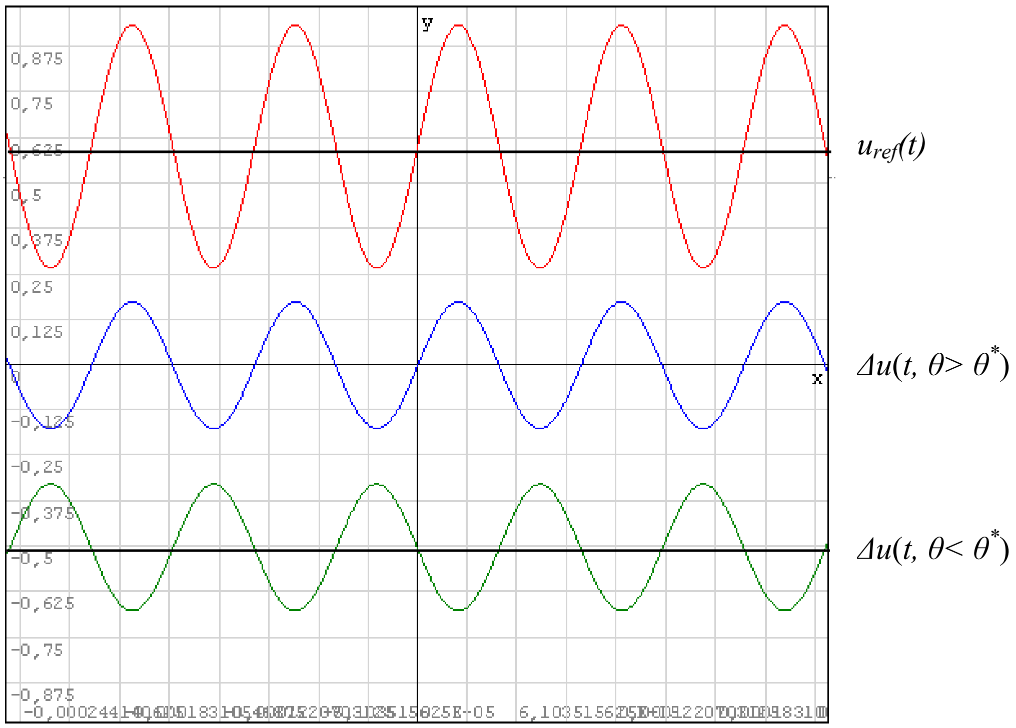

Figure 2: The difference signal $\Delta u(t,)$ für $\theta> \theta^*$ und $\theta< \theta^*$ in comparison to $u_{ref}(t)$

The mathematical background of the procedure is as follows:
The sinus signal of the resolver is multiplied by **cos** θ^* and the cosinus signal by **sin** θ^*. The difference then delivers the signal

$$\Delta u(t) = k \cdot u_0 \left(\sin\theta \cdot \cos\theta^* - \cos\theta \cdot \sin\theta^*\right) \cdot \sin\omega t = k \cdot u_0 \cdot \sin\left(\theta - \theta^*\right) \cdot \sin\omega t \qquad (3)$$

This signal $\Delta u(t)$ now depends on the difference of the angles θ- θ^* which modulates the amplitude of the sine wave. Two cases have to be distinguished:

1. $\theta > \theta^*$: The difference signal $\Delta u(t)$ is in phase with the sine wave $u_{ref}(t)$.
2. $\theta < \theta^*$: The difference signal $\Delta u(t)$ is in opposite phase.

This relationship is shown in figure 2. If the angle θ^* is to approach the value of θ, then in case 1 it obviously has to be increased, while it has to be decreased in case 2. The amplitude of the signal can serve as measure for the change which is necessary. It is proportional to the rectified value. In case 2 the phase has to be taken in account by a negative rectified value, which is achieved by a phase sensitive rectification. Mathematically this is done by multiplying $\Delta u(t)$ with the sign function of $u_{ref}(t)$. The phase sensitively rectified signal is now

$$u_=(t) = \mathbf{sign}\left[u_{ref}(t)\right] \cdot k \cdot u_0 \cdot \mathbf{sin}\left(\theta - \theta^*\right) \cdot \mathbf{sin}\ \omega t \qquad (4))$$

By the multiplication with the sign function of $u_{ref}(t)$ the half waves of the difference signal $\Delta u(t)$ are multiplied with +1 oder –1 in such a way, that the resulting half waves are positive in the case of $\theta > \theta^*$ and negative in the case of $\theta < \theta^*$. The average value of the rectified signal is therefor positiv in case 1 and negative in case 2. These results are shown in figure 3.

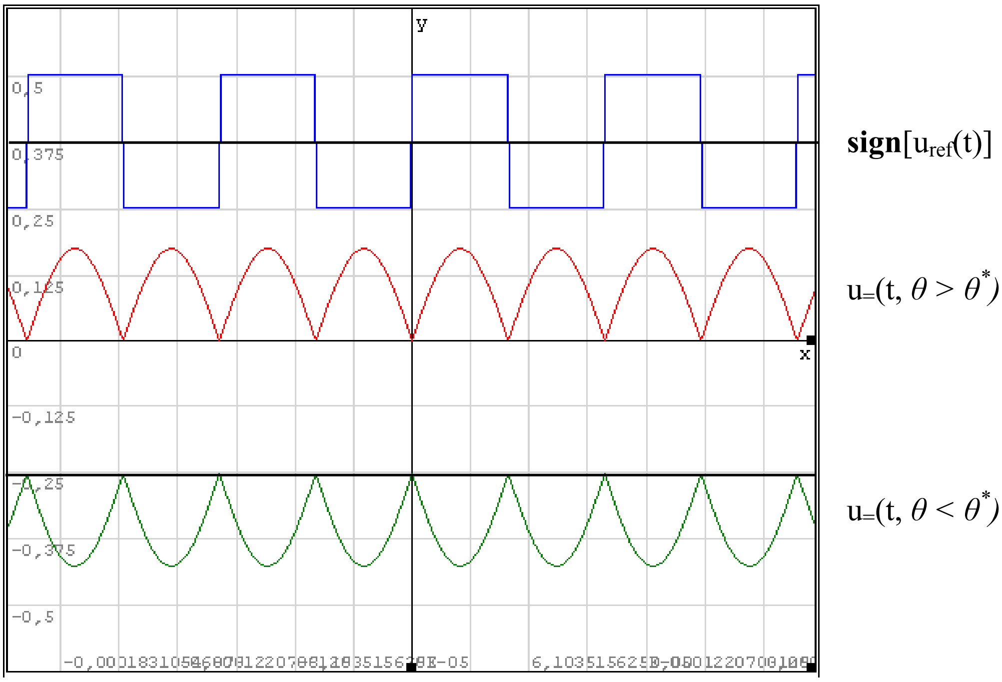

Figure 3: Sign function and phase sensitively rectified signals.
(not to scale)

The rectified signal is smoothed by a low pass filter and then fed through to two integrators in series. The second one produces the estimated value of θ^*, which then can be used in eq. 3.

1. Integrator with a proportional part: $u_I(t) = \int \overline{u_=(t)}\, dt + K_p \cdot \overline{u_=(t)}$ (5)

2. Integrator: $\theta^* = K_I \int u_I(t)\, dt$ (6)

A double integration is necessary, if no persistent errors are tolerated while the measured variable is changing by a ramp (tracking controller of type 2). In order of the control loop to remain stable the first integrator needs a proportional part in parallel.

Functional Plan

The eqs. 3 to 6 are shown as interrelated functional blocks in figure 4. The separate blocks have to be implemented by electronic circuits. In advance it is possible to check, if the arrangement in principle is able to fulfill the requestet task.

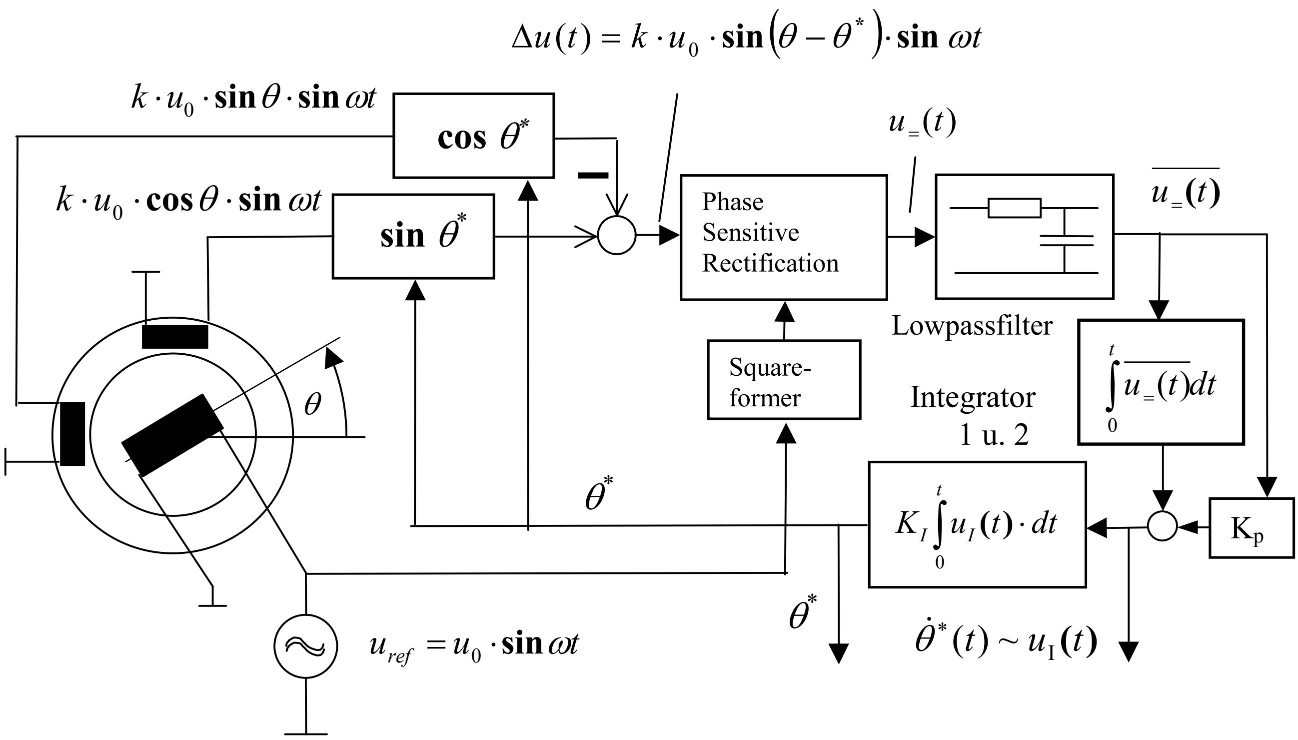

Figure 4: Functional Blocks of a Resolver with Tracking Control

The input signal to the system is the angle of rotation θ of the resolver as measured variable. The output is the angle θ^* as measured value. This is to match the measured variable at all times with an error as small as possible. As there is no information available about the measured variable at power up, large deviations can occur at that moment. Therefor it is interesting to know, if the procedure will work under these condition. The test on some pairs of values is presented in table 1.

θ	θ^*	$\theta - \theta^*$	sin ($\theta - \theta^*$)	**Result**	**Valuation**
0	0	0	0	adjusted	+
90	0	90	1	θ^* is increased	+
179	0	179	0,0175	θ^* is increased	+
180	0	180	0	**no reaktion**	**-**
181	0	181	-0,0175	θ^* is decreased	+
270	0	270	-1	θ^* is decreased	+
359	0	359	-0,0175	θ^* is decreased	+

Tabel 1: System reaction to an selection of angle pairs

The angle of rotation is defined on the range $0 \le \theta \le 360°$, which can be passed through cyclically, meaning after 360° follows 0° again. As a naive assumption the estimated value θ^* is supposed to be zero after power up. Then with regard to table 1 proper functioning can be expected with the exception of $\theta - \theta^* = 180°$. With this combination of values no correction signal will be produced as long as the resolver does not rotate. This can be avoided if θ^* gets an initial value different from zero in this case which is easy to detect. If the initial difference is large, the time for adjustment can be large too. This has eventually to be considered. It will be addressed again later.

Dynamic Model of the Tracking Filter

For the development of a simple, dynamic model it suffices to make use of the slow processes in the system. These are those blocks in the functional plan, which are not dependend on the sine wave, which gets illiminated by the rectification. The smoothing low pass filter and the integral elements form the slow elements of the system.

The rectified value before the low pass filter is

$$u_{=}(t) = k \cdot u_0 \cdot \mathbf{sin}\left(\theta(t) - \theta^*(t)\right) \tag{7}$$

As the behavior with respect to $\theta^* \to \theta$ shall be investigated, the sine function can be linearized and be replaced by its argument leading to the following approximation:

$$\tilde{u}_{=}(t) = k \cdot u_0 \cdot \left(\theta(t) - \theta^*(t)\right) \tag{8}$$

The dynamic model can be represented by the frequency responses of the separate blocks. The Laplace representation of the signals is given here by the bold face symbols, for example

$$L[\theta(t)] = \boldsymbol{\theta}(s) \tag{9}$$

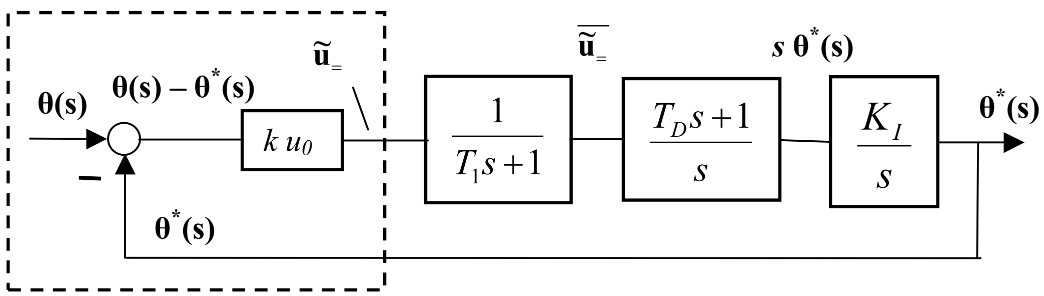

Figure 5: Linearised Dynamic Model

Figure 5 shows the model. Eq. 8 is represented in the dashed frame. It follows the low pass filter of first order with time constant T_1 for smoothing the signal. The output is fed to the first integrator. It contains a derivative term with time constant T_D. Then the second integrator follows with a gain factor K_I. The output represents the measured value $\boldsymbol{\theta}^*$.

Figure 5 describes a control loop which automatically keeps the measured value $\boldsymbol{\theta}^*$ on track of the measured variable $\boldsymbol{\theta}$. As has been mentioned above the control loop is a tracking controller of type 2 that is able to eliminate deviations, also if the measured variable is not constant but changing with constant velocity. The following investigation will show that this goal of design is reached in fact. To show this the transfer function of the open control loop is useful:

$$F_0(s) = \frac{k \cdot u_0}{T_1 \cdot s + 1} \cdot \frac{T_D \cdot s + 1}{s} \cdot \frac{K_I}{s} \tag{10}$$

The control error

$$\mathbf{\Delta\theta}(\mathrm{s}) = \mathbf{\theta}(s) - \mathbf{\theta}^*(s) \qquad (11)$$

as function of $\mathbf{\theta}$(s) follows from figure 5 in the form

$$\mathbf{\Delta\theta}(s) = \frac{1}{1+F_0(s)} \cdot \mathbf{\theta}(s) = \frac{1}{1+\dfrac{k \cdot u_0}{T_1 \cdot s+1} \cdot \dfrac{T_D \cdot s+1}{s} \cdot \dfrac{K_I}{s}} \cdot \mathbf{\theta}(s)$$

$$\mathbf{\Delta\theta}(s) = \frac{(T_1 \cdot s+1) \cdot s^2}{T_1 \cdot s^3 + s^2 + T_D K_I k \cdot u_0 \cdot s + K_I k \cdot u_0} \cdot \mathbf{\theta}(s) \qquad (12)$$

Now $\mathbf{\theta}$*(s)* is set to a ramp function and then it is checked, if the steady state error will vanish for large times. The Laplace transform of a ramp function is

$$\mathbf{\theta}(s) = L\{a \cdot t\} = \frac{a}{s^2} \qquad (13)$$

The control error is now

$$\mathbf{\Delta\theta}(s) = \frac{(T_1 \cdot s+1) \cdot s^2}{T_1 \cdot s^3 + s^2 + T_D K_I k \cdot u_0 \cdot s + K_I k \cdot u_0} \cdot \frac{a}{s^2} \qquad (14)$$

After reducing s^2 the final value theorem of the Laplace transform [6] can be applied. This means:

$$\lim_{t \to \infty} \Delta\theta(t) = \lim_{s \to 0} s \cdot \mathbf{\Delta\theta}(s) \qquad (15)$$

The limit of eq. 14 delivers the result

$$\lim_{s \to 0} s \cdot \mathbf{\Delta\theta}(s) = s \cdot \frac{(T_1 \cdot s+1)}{T_1 \cdot s^3 + s^2 + T_D K_I k \cdot u_0 \cdot s + K_I k \cdot u_0} \cdot a = 0 \qquad (16)$$

With respect to eq. 15 the steady state error disappears. The measured value therefor tracks the measured variable without error while it is changing uniformly. In trasient periods an error still will occur. For stability reasons it is necessary that all powers of s are present in the denominator of eq. 12. Then stability of the control loop can be achieved by a proper choice of the parameters. The values of these parameters are still to be determined.

About Designing the Electronics

The function plan of figure 4 has been realised in practice many times in the form of integrated circuits. Exemplarily the following literature be mentioned [7, 8, 9, 10]. Without digging into details too much the options will be discussed by which the system can be realised by electronic means. The scheme of such a realisation is shown in figure 6.

Input signals of the circuit are the resolver signals

$$u_1 = k \cdot u_0 \, \mathbf{sin}\, \theta \cdot \mathbf{sin}\, \omega t \qquad (17)$$

$$u_2 = k \cdot u_0 \, \mathbf{cos}\, \theta \cdot \mathbf{sin}\, \omega t \qquad (18)$$

They are to be multiplied by $\mathbf{sin}\theta^*$ or $\mathbf{cos}\theta^*$ respectively. A request of the applications is, that the angle θ^* shall be available as binary value of high resolution. The circuitry is therefor known as *Resolver-to-Digital-Converter (RDC)*. The binary value is formed with the aid of an Up/Down-Counter which is driven by a voltage controlled oscillator (VCO). The VCO is an interface between the analog part of the circuit and the digital part. The input signal is an analog voltage, while the output delivers a puls train whose frequency is near to proportional to the input voltage. A constant input voltage will count up the counter uniformly. This resembles the step response of an analog integrator and has the same effect. Figure 7 shows this step response.

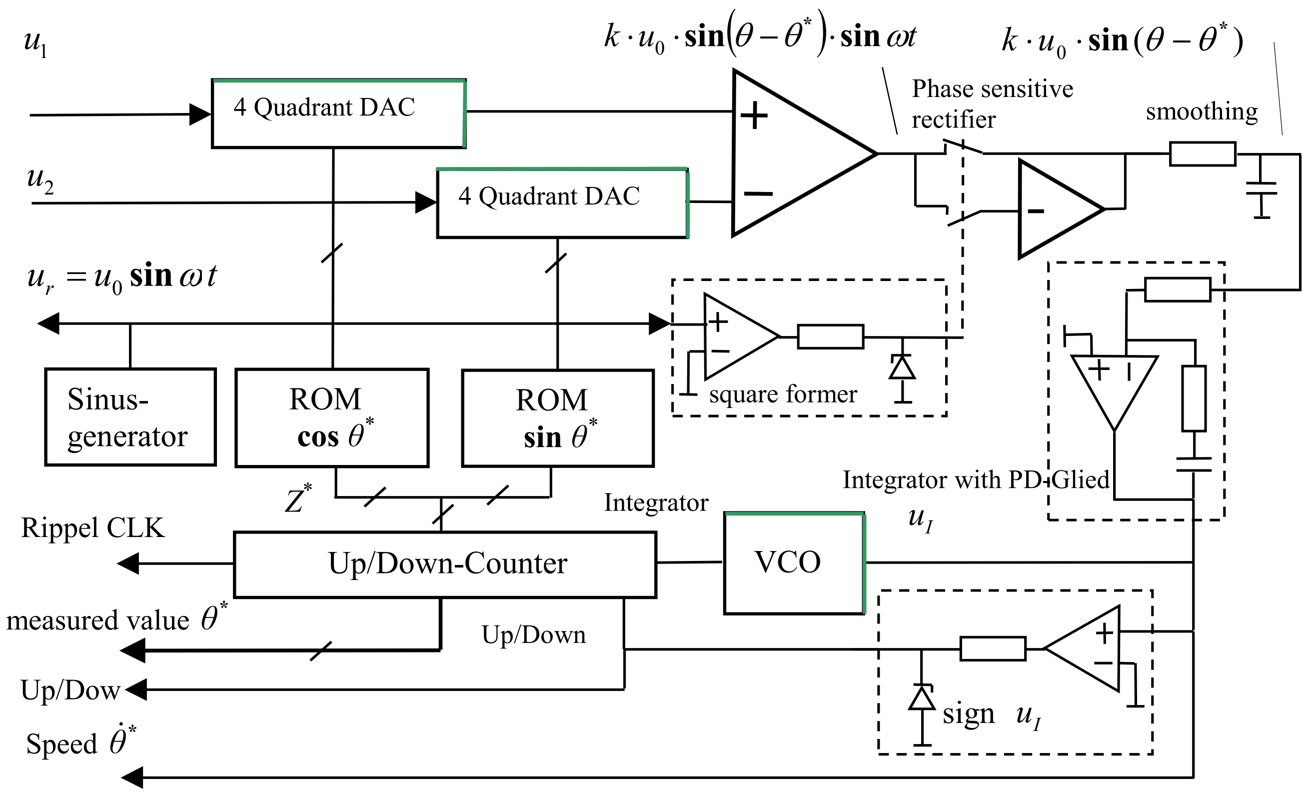

Figure 6: Scheme of the Resolver-to-Digital-Converter (RDC)

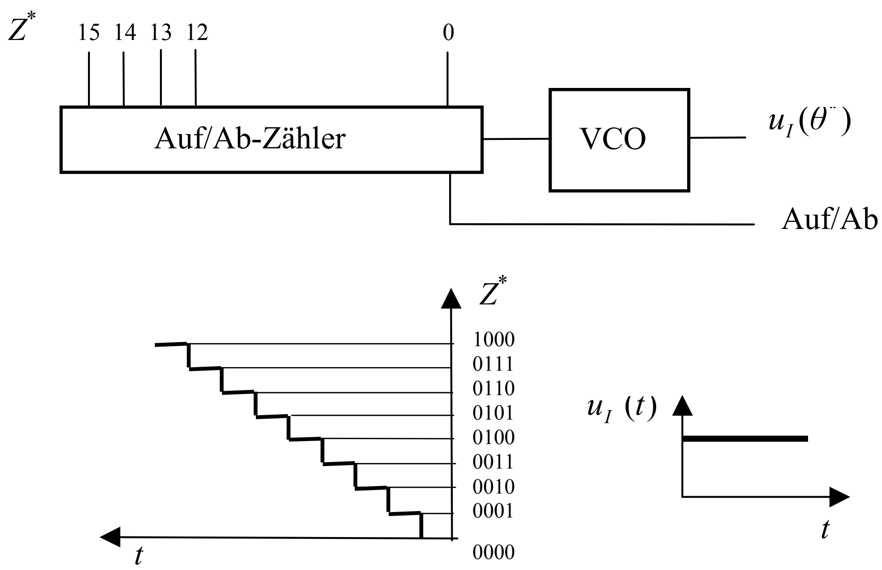

Figure 7: VCO with Up/Down-counter and step response

If the input voltage of the VCO gets negative, then the counter has to count down. The direction of counting is determined by the control signal *Up/Down*. The binary number Z^* is the digital value of the angle θ^*. This binary value is used to address two ROMs which contain the sinus and cosinus values for one period. These values are fed in two multiplying Digital-to-Analog-Converters (DAC), which can multiply in 4 quadrants. The DACs expect the data in Binary-Offset-Code shown in tabel 1, which distinguishes positive und negative values, e.g. [11].

Binär-Offset-Code	**Analog values of the**
MSB LSB	**DACs**
1111 1111 1111 1111	+10 V *(32767 / 32768)
1000 0000 0000 0001	+10 V *(1 / 32768)
1000 0000 0000 0000	0 V
0111 1111 1111 1111	-10 V *(1 / 32768)
0000 0000 0000 0000	-10 V *(32767 / 32768)

Tabel 1: Binär-Offset-Code with 16 Bit

The question of the time needed to adjust the measured value in the worst case can now be quantified. θ^*= 180° would mean a counter value of 32767. If the pulse train of the VCO had a frequency of 10 kHz, then starting from zero it would need 3,27 s to hit the proper value. This shows that the speed of adjustment needs to be considered.

The DACs are interfaces between the analog and the digital part of the circuit simlar to the VCO. They multiply the analog signals of the resolver by the digital sinus and cosinus values. The outputs of the DACs are analog signals. They get subtracted from each other and then become rectified in the phase sensitive way. For this purpose the reference signal is transformed to a unipolar square wave which switches the difference signal through directly while it is in high state and invertedly if it is in low state.

The rectified signal then passes through the low pass filter to the first integrator, which contains a derivative term for stability reasons (see eq. 5). The outputsignal of the first integrator is the time derivative of the angel signal θ^* and therefor has the quality of a speed signal. It can substitut a seperate speed sensor which is widely used in servo control systems. This is an additional feature of the RDCs.

The output of the first integrator controls the voltage controlled oscillator (VCO). It delivers a puls train with a frequency proportional to the input voltage, e. g. [12]. As the puls train does not have a sign, the counting direction of the counter is determined by the *Up/Down*-signal which is derived from the input of the VCO.

The counter value represents the measuring value θ^* in binary form. It is the output of the RDC. Additional output signals are the speed signal and the *Ripple CLK* including the *Up/Down-signal* of the counter which allow the counting of several revolutions. This is useful with resolvers having more than one pair of coils or with inductosysns [13].

Results

The resolver and the resolver-to-digital-converter together form a mechatronical system. The development however took place a long time before the notion of „Mechatronics“ came into use. The resolver is a good example that by the cooperation of different technical disziplines a system with outstanding qualities can be created. The core is the electromechanical part which codes the angle of rotation precisely into the amplitudes of two sinus waves (figure 1). The goal of the design for the mechanics is a concentric rotor without baring play and robust structure. The operation of the resolver can take place in rough envirenment far away from the susceptable elektronic circuits without decreasing the measuring precision.

The calculation of the angle of rotation requests basic mathematical kwoledge of the equations 1 to 4. Later solid knowledge of control engineering is required for the arrangment of the functional plan of figure 4 as well as for the layout of the dynamic model for the tracking controller in the sense of eqs 7 to 16.

The focus of the development, however, is clearly in the field of electronics which has to implement the functional plan by a circuitry of high precision. Even if it is possible to resort to a couple of integrated circuits today [11, 12, 14, 15], this is a task of experienced specialists. The industrial acceptance of the resolver was only reached after the resolver-to-digital-converter was available in integrated form.

From today's view the resolver is an angle measuring sensor having mechanical and electronical components which are excellently adjusted to each other and therefor together show all properties of a nice mechatronical system. This of course is also true for other rotary encoders. However the development has lasted a long time and has been done at different places by several specialists. And no one of them had an idea of mechatronics. Mechatronics is therefor not a new field, but a new sight on the relavant fields in order to make such developments more efficient.

Outlook

The development of resolvers is not completed. Beside the devices with a rotor coil new kinds have shown up which rely on rotors with variable reluctance [2, 16]. Another variation is the inductosyn [13], which uses meander inductors in the plane. Regarding the resolver-to-digital-converters one can observe solutions on the basis of microcontrollers [8, 17].

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